Analysis Software for The Cluster Mission: The Spatial Gradient of Density

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ABSTRACT

All physically observable field parameters within the magnetosphere, such as particle densities and electric and magnetic fields, vary in both space and time. Understanding the underlying physical processes requires knowledge of both temporal and spatial variations of these parameters. Analysing data acquired simultaneously from two satellites allows determination of the component of the spatial gradient in the direction of their separation. However, to determine all three components of the spatial gradient, data from at least four spacecraft is required. This is the main objective of the Cluster mission: four identical spacecraft forming a tetrahedron in space.

We present here the least square method as a means of calculating the spatial gradient of plasma density from data gathered using Cluster's WHISPER (Waves of High frequency and Sounder for Probing of Electron density by Relaxation) experiment. Also presented here is software developed for the calculation and visualisation of the density gradient. The software plots the trajectory of the centre of mass of the four spacecraft, showing the density measured at each of the four spacecraft and the gradient density at the centre of mass along the trajectory.

This work is included in preparation of the Cluster mission, developing methods and tools for the analysis of the subsequent multipoint measurements.

KEYWORDS

Cluster, Density, Gradient, Volumetric Tensor, WHISPER.

INTRODUCTION

The Belgian Institute for Space Aeronomy (IASB-BIRA) is involved in the ESA Cluster II mission [*Escoubet et al.*, 1997]. Composed of four identical spacecraft, Cluster II will study the small-scale plasma structure in space and time in key plasma regions (solar wind, bow shock, magnetopause, polar cusp, magnetotail and auroral zone). Each satellite contains 11 instruments to measure electric and magnetic fields, electron and ion distribution functions, and electromagnetic waves in 3-dimensions. Analysing data acquired simultaneously from the four spacecraft allows the three components of the spatial gradient of these parameters to be obtained.

IASB-BIRA participates in two Cluster II experiments:

a) WHISPER (Waves of High frequency and Sounder for Probing of Electron density by Relaxation); Principal Investigator Dr. P. Décréau (LPCE-CNRS, Orléans, France); IASB-BIRA Co-Investigator Prof. J. Lemaire. This experiment is described in [*Décréau et al.*, 1997].
b) STAFF (Spatio-Temporal Analysis of Field Fluctuations); Principal Investigator Dr. N. Cornilleau-Wehrlin (CETP, Vélizy, France); IASB-BIRA Co-Investigator Dr. M. Roth. The participation in STAFF is described by [*Roth et al.*, 2001].

Under the support of ISSI (International Space Science Institute), a working group on 'Advanced Analysis Methods for Data from Clusters of Spacecraft' was created to investigate methods and tools for the analysis of multipoint measurements. A ISSI Scientific Report [*Paschmann and Daly*, 1998] presents two methods for the calculation of the spatial gradient of a quantity (scalar or vector): the homogeneous least square method (developed by Dr. C. Harvey, CESR, Toulouse, France) and the linear barycentric method (developed by Dr. G. Chanteur, CETP, Vélizy, France). Results using the two methods are largely equivalent, however, it is the least square method that we have applied to the analysis of plasma density measurements from the WHISPER experiment.

WHISPER is an intermittent transmitter/receiver that can be operated either in passive (receive only) or active mode. In active mode, it emits short pulses (1 millisecond) of sine wave, while other antennas located within the instrument measure the transmitted electric field. The electric field measurements allow identification of the plasma resonances. Analysis of the resonance positions within the measured electric field spectra provides the absolute value of the total electron density within the range 0.2 to 80 electrons per cubic centimeter. In this mode, as the instrument is not influenced by spacecraft potential fluctuations, WHISPER can measure the plasma density very accurately, making the determination of the spatial gradient of the plasma density, by combining measurements from all four Cluster spacecraft, of great interest.

The first part of this paper will give a brief summary of the least square method. The second part will describe software used for the calculation and visualisation of the density gradient. Finally, some examples of analysing 'test' Cluster data are presented.

THE LEAST SQUARE METHOD

Introduction

The least square method, used to determine the spatial gradient of a parameter using data acquired simultaneously from four or more spacecraft is described in chapter 12 of [*Paschmann and Daly*, 1998].

The method is based on the principle that the spatial gradient can be expressed in terms of the inverse of a symmetric tensor formed from relative positions of the spacecraft. It is shown that this tensor describes basic geometrical properties of the polyhedron defined by the four or more spacecraft, specifically, its characteristic size (mean square thickness) and orientation in three mutually orthogonal directions. These six geometrical parameters completely define the symmetric tensor, and so contain all the geometrical information required to determine the spatial gradient by the least square method.

In the special case of four spacecraft, the product of the three characteristic dimensions is exactly three times the volume of the tetrahedron formed by the spacecraft. Consequently the symmetric tensor is referred to as the volumetric tensor. Through its eigenvalues and eigenvectors the volumetric tensor provides a description of the geometry of the polyhedron. The eigenvalues can be used to define three geometrical parameters; the characteristic size L, the elongation E and the planarity P. The eigenvectors can be used to define the direction of elongation $\mathbf{e}_E$ and planarity $\mathbf{e}_P$. These parameters completely describe the physically significant geometrical information of the tetrahedron (chapter 13, [*Paschmann and Daly*, 1998]).

Brief Summary of the Method

It has been shown that for four spacecraft α (α= 1, 2, 3 or 4), if r_α^i and r_α^j are the positions, measured simultaneously, of each of the spacecraft in the *x, y* and *z* directions (*i,j = x,y* or *z*) and r_b^i and r_b^j the mean values, over all α, of r_α^i and r_α^j (i.e. the position of the centre of mass of the four spacecraft in the *x, y* and *z* directions), the volumetric tensor R_{ij} may be written as

$$R_{ij} = \frac{1}{4}\sum_{\alpha=1}^{4}\left(r_\alpha^i - r_b^i\right)\left(r_\alpha^j - r_b^j\right)$$

If n_α is the value of density *n* measured simultaneously on each of the spacecraft α, it can be shown that the least square estimation of the density gradient is

$$\frac{\partial n}{\partial r_i} = \frac{1}{2}\frac{1}{4^2}\sum_j\left[\sum_{\alpha=1}^{4}\sum_{\beta=1}^{4}\left(n_\alpha - n_\beta\right)\left(r_\alpha^j - r_\beta^j\right)\right]\times R_{ji}^{-1}$$

where R_{ji}^{-1} is the inverse of the volumetric tensor.

We can express R_{ji}^{-1} in terms of the geometrical parameters L, E, P and of the directions of elongation and planarity $\mathbf{e}_E$ and $\mathbf{e}_P$

$$R_{ji}^{-1} = \frac{4}{L^2}\left[e_{E_j}e_{E_i} + \frac{1}{(1-E)^2}e_{L_j}e_{L_i} + \frac{1}{(1-P)^2(1-E)^2}e_{P_j}e_{P_i}\right]$$

where $\mathbf{e}_L$, is the third, mutually orthogonal, direction: $\mathbf{e}_L = \mathbf{e}_P \times \mathbf{e}_E$.

The three parameters L, E, and P, the two directions $\mathbf{e}_E$ and $\mathbf{e}_P$ and the plasma density, obtained from the WHISPER instrument, are available on CD-ROM delivered to the experiment Co-Investigator's, and also on CDF format from the Cluster II data centre.

Estimation of the Error

The 6x6 orbital covariance matrix gives the estimated error of each satellite position and momentum. The WHISPER instrument evaluates an error of the density measurement. This data is also available on the CD-ROM or from the data centre. Under some circumstances it will not be possible to determine the gradient, for example, when the satellites are coplanar (P=1) or collinear (L=1).

SOFTWARE AND SOME EXAMPLES

A *Matlab* program was written to read the Cluster data in CDF format, calculate the resulting density gradient and plot the results. The software allows the trajectory of the centre of mass of the four spacecraft, the density measured at each of the four spacecraft, and the gradient density, to be plotted as a function of time.

Figures 1 to 3 show an example of these plots for the special case of a density profile following the Harris density model

$$n\left(r_x, r_y, r_z\right) = N_0 + \frac{N_x}{\cosh^2\left(r_x/L_x\right)} + \frac{N_y}{\cosh^2\left(r_y/L_y\right)} + \frac{N_z}{\cosh^2\left(r_z/L_z\right)}$$

where n is the density at position (r_x, r_y, r_z), with $N_{x,y,z}$ the maximum of the density profile and $L_{x,y,z}$ the density profile thickness in the x, y and z directions. The figures show simulated orbital data of Cluster combined with this density profile (without z-axis dependence [N_z= 0]).

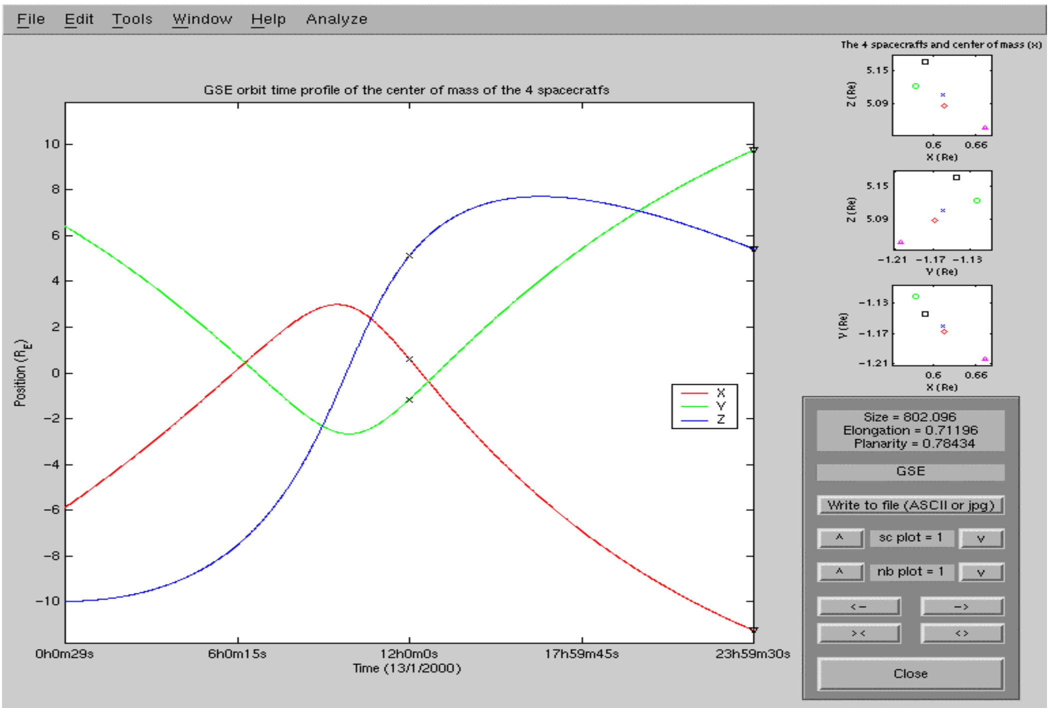

Figure 1: Trajectory of the centre of mass of the 4 Cluster spacecraft over a 24-hour time period.

Figure 1 shows the trajectory of the centre of mass of the four spacecraft as a function of time. The time period is 24 hours. Shown on the top right hand side of the plot are the positions of the spacecraft and of the centre of mass in the middle of the trajectory (shown as crosses in the main plot) in three different GSE reference frames *XY*, *YZ* and *XZ*. Values of the geometrical parameters L (labelled Size), E (labelled Elongation) and P (labelled Planarity) are given at this middle position. It is also possible to have this trajectory projected onto each of the three reference frames.

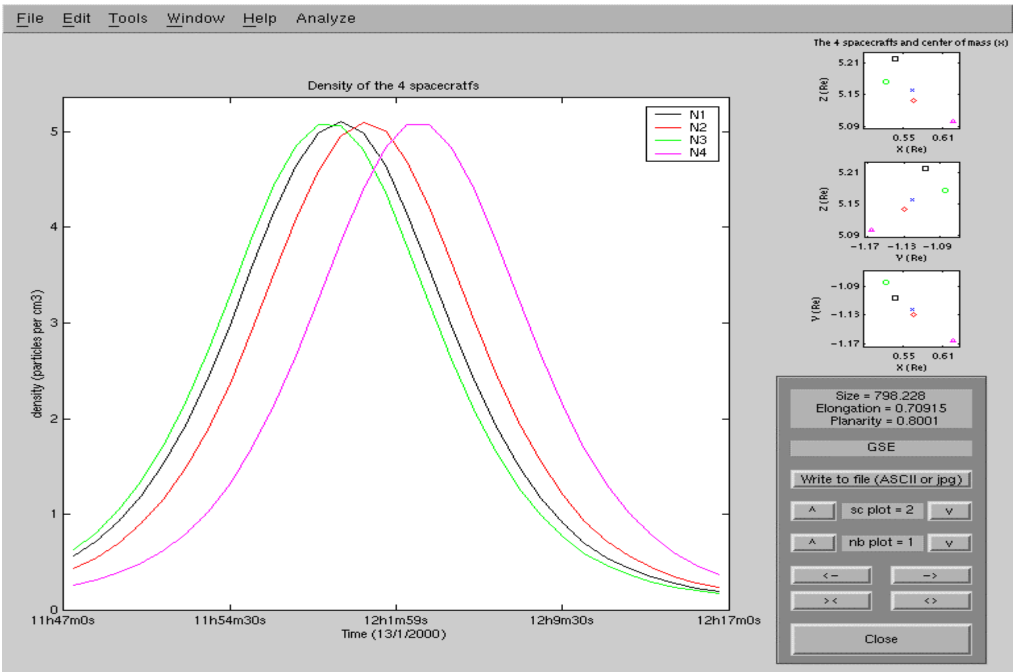

Figure 2: Density profiles measured by the 4 Cluster spacecraft over a 30-minute time period.

Figure 2 shows the density *n* measured by each of the spacecraft as a function of time. The time period is variable, in this case 30 minutes of the 24-hour period of Figure 1. Again, spacecraft positions and values of L, E and P are shown while in the middle of the trajectory.

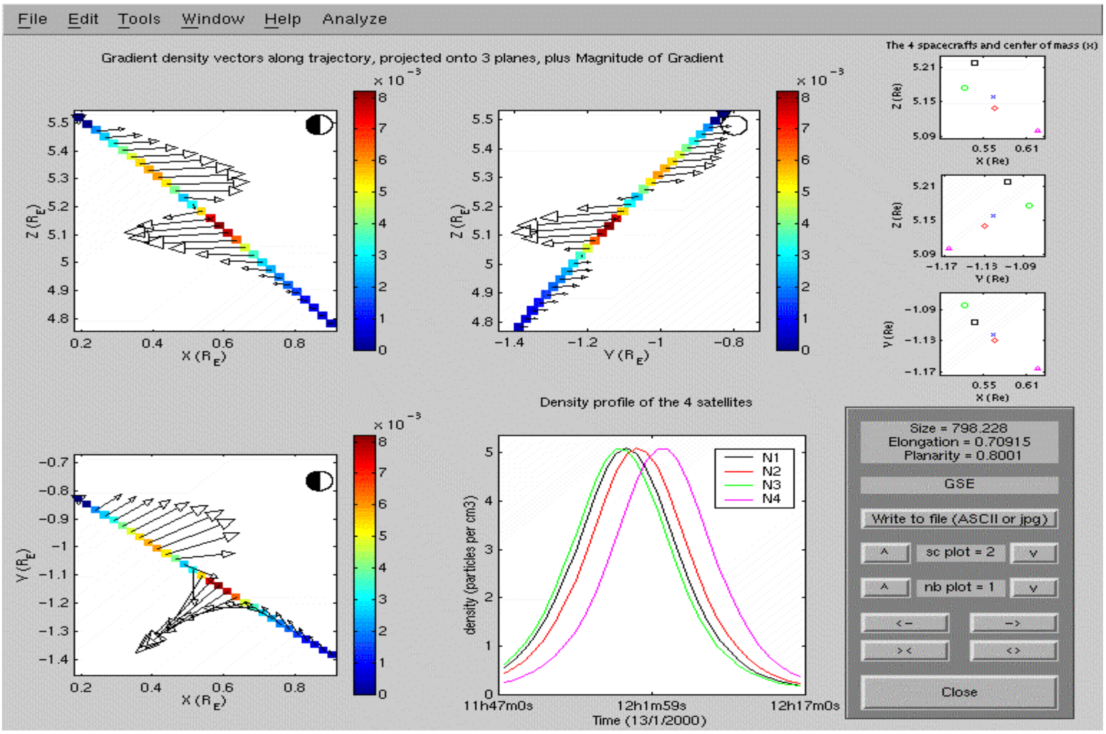

Figure 3: Density gradient of the 4 Cluster spacecraft along the trajectory of the centre of mass projected onto the XZ, YZ and XY planes and for a 30-minute time period.

Figure 3 shows the density gradient profile calculated, using the least square method, at the centre of mass of the four spacecraft and plotted along the trajectory of the centre of mass. The density gradient is projected onto the *XY*, *YZ* and *XZ* reference frames, in each case both the magnitude and components of the gradient are shown. It is also possible to plot the gradient without the magnitude or to plot it in only one reference frame. The results of Figure 3 are consistent with the input density profile having no z-axis dependence. Examining the upper XZ and YZ plots, it can be seen that, as expected, there is no density gradient in the z-direction, whereas, the lower XY plot shows a density gradient in the x and y-directions.

CONCLUSION

With Cluster, we have, for the first time, the possibility of making simultaneous measurements onboard four spacecraft located in different regions of the magnetosphere. The development of software for the analysis of the subsequent multipoint measurements, enabling us to reveal gradient profiles of vector and scalar quantities, is of great interest.

The density gradient software discussed in this paper is based on the least square method, applied to plasma density measurements made by Cluster's WHISPER experiment. The resulting software will be used to evaluate the density gradient in critical regions such as, for example, the magnetopause. The magnetopause is a region where density gradient profiles will provide much information on the structures observed, structures which have been hard to interpret based on observations made by two satellites (see, for example, the interpretation of transient phenomena observed near the magnetopause by Interball-Tail and Magion-4 satellites [*Vaisberg et al.*, 1998] and [*De Keyser et al.*, 2001]).

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